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Numerical Modeling of Wave Processes in Rocks by the Grid-Characteristic Method

A. V. Favorskaya[a, b, c, *] **and I. B. Petrov**[a, b, **]

[a] *Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow oblast, 141701 Russia*
[b] *Scientific Research Institute of System Analysis, Russian Academy of Sciences, Moscow, 117218 Russia*
[c] *Educational, Scientific and Production Center,*
Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow oblast, 141700 Russia
**e-mail: aleanera@yandex.ru,*
***e-mail: petrov@mipt.ru*

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Abstract—The aim of this work is to study the spatial dynamic of wave propagation in rock formations, taking into account ravines and caverns. The dynamics of the seismic and acoustic waves generated by explosions of different types is investigated using seismograms measured by several reception lines. The research uses the numerical experiments using the full-wave joint simulation of acoustic and seismic wave propagation in heterogeneous mixed acoustic and linear-elastic media. The grid-characteristic method is used to obtain the mathematically and physically correct description of spatial dynamic wave processes taking the boundary and contact surfaces, including the interfaces between the linear-elastic and the acoustic environments, into consideration. The influence of the type of explosion on the spatial dynamic wave patterns and seismograms is analyzed for the cases of horizontal and vertical reception lines. The dependences of spatial dynamic wave patterns and seismograms recorded by the horizontal and vertical reception lines on the distance of the karst caverns from the ravine are studied. The basic types of waves that different types of explosions generate in the rock formations, ravines, and caverns are investigated. The basic laws that characterize the emerging wave patterns and their influence on seismograms are found.

Keywords: spatial dynamic wave processes, numerical simulation, grid-characteristic method, rock formations, seismograms

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1. INTRODUCTION

This study is devoted to examining spatial dynamic wave processes and seismograms originating in the solution of the problem on seismic and acoustic wave propagation from a source of explosion in rock formations containing ravines and karst caverns. The influence of the type of explosion on the spatial dynamic wave patterns and seismograms obtained from the horizontal and vertical reception lines is analyzed. The influence of the distant location of karst caverns in relation to the ravine on the spatial dynamic wave patterns and seismograms obtained from horizontally and vertically arranged reception lines is analyzed.

The discrete element method (DEM) or a variant of it, the destructive deformation analysis (DDA) and couple-mode approach, are conventionally used for the numerical simulation of the rock formation mechanics [1, 2]. In [3, 4], a simple rigid-block model is developed to analyze cavern and tunnel stability in the surrounding rock formation. In [5], the DYNA2D complex code (finite element method) is applied to simulate the seismic response from a tunnel in the surrounding mountain formation. In [6], the finite element and the rigid-block models are used for the numerical simulation of extending the earth's waves in caverns and the surrounding hills. In [7], the finite element (FE), finite difference (FD), and arbitrary difference precise integration (ADPI) methods for seismic direct simulation have been developed and implemented to investigate the propagation of seismic waves in composite geological underground structures and reservoirs in China.

This study has been carried out by the joint full-wave simulation. The system of equations describing the behavior of an infinitesimal volume of an isotropic linear-elastic medium and the system of equations

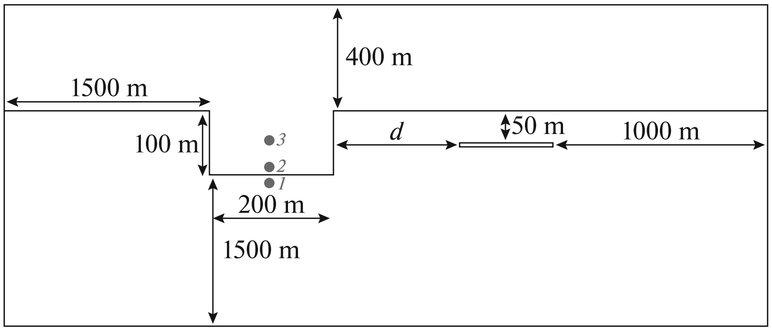

Fig. 1. A sketch of a model.

describing acoustic spatial dynamic wave processes [8] are solved jointly using the grid-characteristic method [9–13]. This method has allowed us to perform the full-wave simulation in heterogeneous media considering the contact and boundary conditions, including the contact condition at the boundaries between the linear-elastic and the acoustic media [8, 14].

2. FORMULATION OF THE PROBLEM

A system (Fig. 1) consisting of a ravine surrounded with rock formations, the airspace inside the ravine and above the rock formations, and a karst cavern [15, 16] measuring 150 m × 5 m filled with air and located at a variable distance *d* from the ravine is considered. The distance *d* is assumed to be 0 m (the karst cavern is directly connected with the ravine), 200 m, and 1000 m. Three types of explosions are considered: an explosion in the center of the ravine immersed in the rock formations (*1*), an explosion at the bottom of the ravine in the center (*2*), and an explosion in the center of the ravine (*3*). The radius of the disturbed region is 2.5 m. The explosion is simulated in the form of an impulse representing one phase of a sine. The amplitude of the disturbance of the relative pressure is 5000 Pa. The sound velocity in the air, 331 m/s, and density of 1.2754 kg/m^3 are used in the calculations. The following properties of the rock formations are considered: the P-wave velocity 3000 m/s, S-wave velocity 1500 m/s, and density 2000 kg/m^3, time step 0.00008 s, and spatial coordinate step 0.25 m; 15000 time steps are considered.

A system reflecting the state of a linear-elastic medium [8, 14, 17, 18] is solved in order to describe space dynamic wave processes in the rock formations. The system describing an acoustic field [8, 14, 19] is solved for the case considering processes in the atmosphere.

At the interfaces between the acoustic and the linear-elastic media, the following contact condition is set [8, 14]:

$$p = -(\boldsymbol{\sigma} \cdot \mathbf{n}) \cdot \mathbf{n}, \tag{1}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n} - \big((\boldsymbol{\sigma} \cdot \mathbf{n}) \cdot \mathbf{n}\big)\mathbf{n} = 0, \tag{2}$$

$$\mathbf{v}^{\mathrm{A}} \cdot \mathbf{n} = \mathbf{v}^{\mathrm{E}} \cdot \mathbf{n}. \tag{3}$$

In (1)–(3), $\mathbf{n}$ is the normal vector external to the linear-elastic layer, in (3) $\mathbf{v}^{\mathrm{A}}$ is the velocity in the acoustic layer, and $\mathbf{v}^{\mathrm{E}}$ is velocity in the linear-elastic layer. Condition (1) stipulates that the normal component of the surface density of forces from the side of a solid body is equal to the pressure in an ideal liquid. Condition (2) requires the tangentional component of the surface density of the forces from the side of the solid body to be equal to zero. Condition (3) sets the equality of the velocity's normal components in an ideal liquid and in a solid body. Nonreflecting conditions are set at the lateral boundaries of the integration domain. A zero initial disturbance is used.

Receivers recording seismograms (seismic traces and the velocity dependences on time) were placed in a karst cavern near the rock boundary along two lines, vertical and horizontal (Fig. 2). The vertical line of reception consisted of 21 receivers positioned 0.25 m apart. The horizontal reception line consisted of 151 receivers spaced at a distance of 1 m from each other (Fig. 2).

The grid characteristic method is used to obtain a compatible numerical solution of the system of equations describing elastic waves and the system of equations describing acoustic waves; the method is described in detail in [8–11, 14]. The grid characteristic method allows constructing the correct numerical

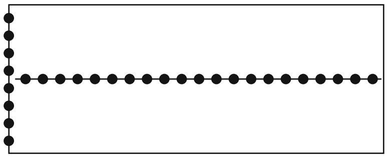

Fig. 2. Arrangement of reception lines in karst cavern.

algorithms to calculate the inner points of the integration domain taking into account its heterogeneous structure and to calculate the boundary points and points lying on the interfaces of two media with different seismic properties.

3. THE OBSERVED WAVE TYPES

After analyzing the spatial dynamic wave fields of velocity and relative pressure, the following types of waves typical for the problem on acoustic and seismic wave propagation in rocky massifs with ravines and caverns caused by various types of explosions have been revealed. In the case when the explosion occurs in the center of the ravine and near the bottom of a ravine, the following types of waves are observed:

1. Primary waves in the acoustic medium of the ravine

2. Secondary waves in the elastic medium of the rock formations surrounding the ravine

2.1. Longitudinal (P-waves) in the elastic medium of the rock formations surrounding the ravine

2.2. Transverse (S-waves) in the elastic medium of the rock formations surrounding the ravine

2.3. The Rayleigh waves in the elastic medium of the rock formations surrounding the ravine.

2.4. The Love waves in the elastic medium of the rock formations surrounding the ravine

2.5. Longitudinal head waves in the elastic medium of the rock formations surrounding the ravine.

3. Secondary waves in the acoustic medium of the ravine

3.1. Secondary waves in the acoustic medium of the ravines caused by P-waves in the elastic medium of the rock formations surrounding the ravine

3.2. Secondary waves in the acoustic medium of the ravine caused by S-waves in the elastic medium of the rock formations surrounding the ravine

3.3. Secondary waves in the acoustic medium of the ravine caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

3.4. Secondary waves in the acoustic medium of the ravine caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

3.5. Secondary waves in the acoustic medium of the ravine caused by longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

4. Secondary waves in the acoustic medium of the atmosphere above the ravine

4.1. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by P-waves in the elastic medium of the rock formations surrounding the ravine

4.2. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by S-waves in the elastic medium of the rock formations surrounding the ravine

4.3. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

4.4. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

4.5. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

5. Secondary waves in the acoustic medium inside the karst caverns in the rock formations

5.1. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by P-waves in the elastic medium of the rock formations surrounding the ravine

5.2. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the S-waves in the elastic medium of the rock formations surrounding the ravine

5.3. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

5.4. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

5.5. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

In the case when the explosion occurs in the rock formations, the following types of waves are observed:

1. Primary waves in the elastic medium of the rock formations surrounding the ravine

1.1. P-waves in the elastic medium of the rock formations surrounding the ravine

1.2. S-waves in the elastic medium of the rock formations surrounding the ravine

1.3. The Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

1.4. The Love waves in the elastic medium of the rock formations surrounding the ravine

1.5. Longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

2. Secondary waves in the acoustic medium of the ravine

2.1. Secondary waves in the acoustic medium of the ravine caused by P-waves in the elastic medium of the rock formations surrounding the ravine

2.2. Secondary waves in the acoustic medium of the ravine caused by S-waves in the elastic medium of the rock formations surrounding the ravine

2.3. Secondary waves in the acoustic medium of the ravine caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

2.4. Secondary waves in the acoustic medium of the ravine caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

2.5. Secondary waves in the acoustic medium of the ravine caused by the longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

3. Secondary waves in the acoustic medium of the atmosphere above the ravine

3.1. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by P-waves in the elastic medium of the rock formations surrounding the ravine

3.2. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by S-waves in the elastic medium of the rock formations surrounding the ravine

3.3. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

3.4. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

3.5. Secondary waves in the acoustic medium of the atmosphere above the ravine caused by longitudinal head waves in the elastic medium of the rock formations surrounding the ravine

4. Secondary waves in the acoustic medium inside the karst caverns in the rock formations

4.1. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by (P-waves) in the elastic medium of the rock formations surrounding the ravine

4.2. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by S-waves in the elastic medium of the rock formations surrounding the ravine

4.3. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the Rayleigh waves in the elastic medium of the rock formations surrounding the ravine

4.4. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the Love waves in the elastic medium of the rock formations surrounding the ravine

4.5. Secondary waves in the acoustic medium inside the karst caverns in the rock formations caused by the longitudinal head waves in the elastic medium of the rock formations surrounding the ravine.

The relative pressure in the linear-elastic medium of the rock formations is $-(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3$ for the three-dimensional case and $-(\sigma_{xx} + \sigma_{yy})/2$ for the two-dimensional case.

In Fig. 3, the wave patterns of the module of velocity and relative pressure are presented at instant 0.2 s corresponding to the case of an explosion in a rock formation and karst cavern adjacent to the ravine.

In Fig. 4, the wave patterns of the module of velocity and relative pressure are presented at the instant of 0.36 seconds corresponding to the case of an explosion in the rock formation and karst cavern located at a distance of 200 m from the ravine.

Fig. 3. Snapshots of coupled elastic and acoustic wavefields corresponding to velocity module and relative pressure for case of explosion in rock formations.

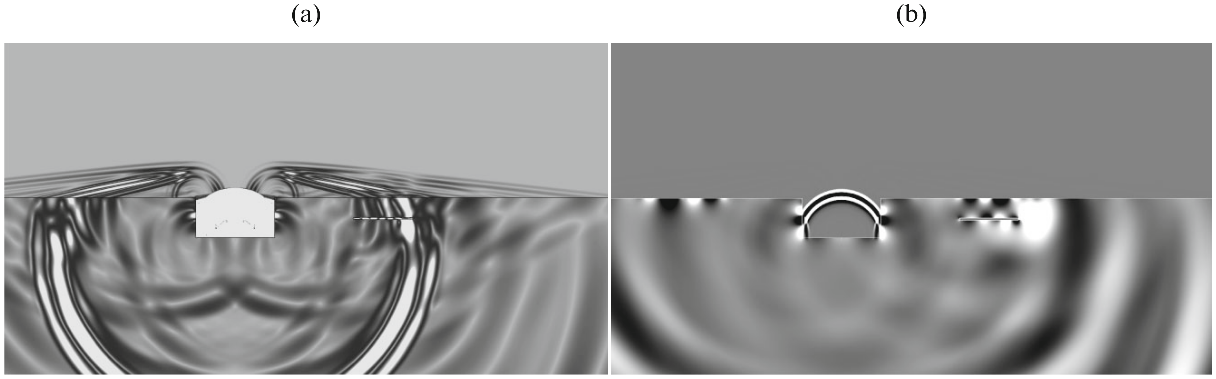

Fig. 4. Snapshots of coupled elastic and acoustic wavefields corresponding to velocity module and relative pressure for case of explosion at bottom of ravine.

In Fig. 5, wave patterns of the module of velocity and relative pressure are presented at the instant of 0.88 seconds corresponding to the case of an explosion in the center of the ravine and karst cavern located at a distance of 1000 m from the ravine.

In Fig. 3a, the nonlinear shade of gray represents the field of the module of velocity, and the white color corresponds to at least 0.1 m/s. In Figs. 4a and 5a, the nonlinear shade of grey represents the field of the module of velocity, and the white color corresponds to a value equal or greater than 0.0001 m/s. In Figs. 3b, 4b, and 5b, the fields of the relative pressure in the scale from white to black ±100 Pa are presented.

4. THE RESULTS OF ANALYZING THE WAVE PATTERN

Having analyzed the spatial dynamic wave patterns, we obtain the following conclusions:

1. In the case when the source of the explosion is positioned in the rock formations, the waves of relative pressure barely spread beyond the limits of the rock formations in the atmosphere. The amplitude of the secondary waves of the relative pressure in the atmosphere generated by the source of the explosion in the rock formations is three orders of magnitude lower than the amplitude of primary waves of the relative pressure originated by the source of the explosion in the rock formations.

2. In the case when the source of the explosion is positioned in the rock formations, the waves of the module of velocity penetrate clearly through the rock–atmosphere interface. The amplitude of the secondary waves of the module of velocity in the atmosphere generated by the source of the explosion in the rock formations is almost the same as the amplitude of the primary waves of the module of velocity originated by the source of the explosion in the rock formations.

3. In the case when the source of the explosion is positioned in the atmosphere, the waves of relative pressure penetrate clearly through the rock–atmosphere interface. The amplitude of the secondary waves of the relative pressure in the rock formations generated by the source of the explosion in the atmosphere

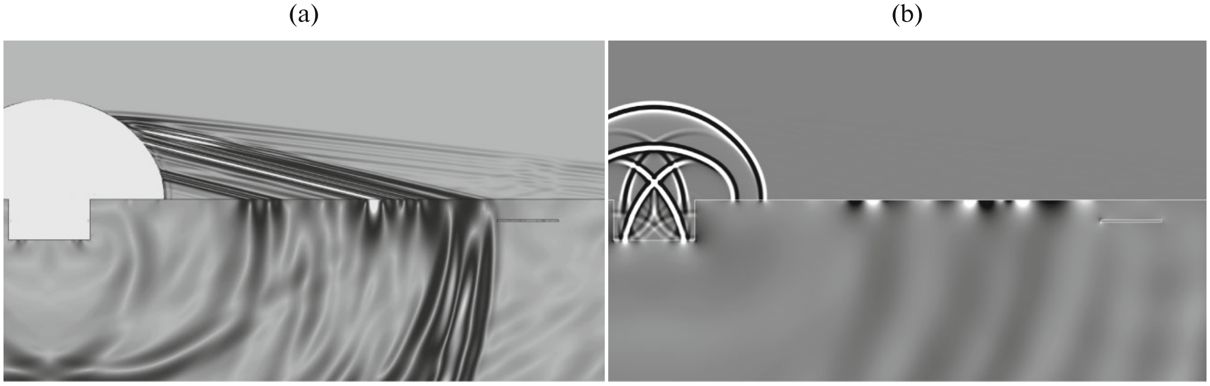

Fig. 5. Snapshots of coupled elastic and acoustic wavefields corresponding to velocity module and relative pressure for case of explosion at center of ravine.

is almost the same as the amplitude of the primary waves of the relative pressure originated by the source of the explosion in the atmosphere.

4. In the case when the source of the explosion is positioned in the atmosphere, the waves of the module of the velocity penetration through the atmosphere–rock interface is rather insignificant. The amplitude of the secondary waves of the module of velocity in the rock formations generated by the source of the explosion in the atmosphere is three orders of magnitude lower than the amplitude of the primary waves of the module of velocity originated by the source of the explosion in the atmosphere.

5. In the case when the source of explosion is positioned in the rock formations, while the karst cavern is directly connected with the ravine, the amplitude of the secondary waves of the relative pressure in the karst cavern is about three orders of magnitude lower than the amplitude of the primary waves of the relative pressure originated by the source of the explosion in the rock formations.

6. In the case when the source of the explosion is positioned in the rock formations, and the karst cavern is not connected directly with the ravine, the amplitude of the secondary waves of relative pressure in the karst cavern is about six orders of magnitude lower than the amplitude of the primary waves of relative pressure originated by the source of the explosion in the rock formations, i.e., the secondary waves are insignificant.

7. In the case when the source of the explosion is positioned in the rock formations and the karst cavern is directly connected with the ravine, the amplitude of the secondary waves of the module of the velocity in the karst cavern is almost the same as the amplitude of the primary waves of the module of the velocity originated by the source of the explosion in the rock formations.

8. In the case when the source of the explosion is positioned in the rock formations and the karst cavern has no direct connection with the ravine, the amplitude of the secondary waves of the module of velocity in the karst cavern is about three orders of magnitude lower than the amplitude of the primary waves of the module of velocity originated by the source of the explosion in the rock formations.

9. In the case when the source of the explosion is positioned in the atmosphere and the karst cavern is directly connected with the ravine, the primary waves of the relative pressure and velocity directly reach the karst cavern without material loss of amplitude.

10. In the case when the source of the explosion is positioned in the atmosphere and the karst cavern is directly connected with the ravine, the amplitude of the secondary waves of relative pressure in the karst cavern is three orders of magnitude lower than the amplitude of the primary waves of relative pressure in the ravine and karst cavern.

11. In the case when the source of the explosion is positioned in the atmosphere and the karst cavern is directly connected with the ravine, the amplitude of the secondary waves of the module of velocity in the karst cavern is three orders of magnitude lower than the amplitude of the primary waves of the module of velocity in the ravine and karst cavern.

12. In the case when the source of the explosion is positioned in the atmosphere and the karst cavern has no direct connection with the ravine, the amplitude of the secondary waves of the relative pressure in the karst cavern is three orders of magnitude lower than the amplitude of the primary waves of the relative pressure in the ravine.

13. In the case when the source of the explosion is positioned in the atmosphere and the karst cavern has no direct connection with the ravine, the amplitude of the secondary waves of the module of velocity in the karst cavern is three orders of magnitude lower than the amplitude of the primary waves of the module of velocity in the ravine.

14. The disposition of the source of the explosion in the atmosphere of the ravine (in the center relative to the ravine or near the bottom) influences only the time of the arrival of the disturbance at the karst cavern.

15. In the case of a karst cavern not connected directly with the ravine, the distance between the ravine and the karst cavern mostly affects the arrival time of the disturbance at the karst cavern.

16. The waves of relative pressure and the waves of the module of velocity propagating inside the karst cavern reach the greatest amplitudes in the case when the karst cavern is directly connected with the ravine, irrespective of the disposition of the explosion; otherwise, the amplitudes of the secondary waves in the atmosphere of the karst cavern are about three to six orders of magnitude lower than the primary wave amplitudes.

5. RESULTS OF THE SEISMOGRAMS ANALYSIS

The analysis of the seismograms obtained using the vertical and horizontal lines of reception shows the following regularities:

1. The seismograms from the horizontal lines of reception are more informative than the seismograms from the vertical lines of reception. This is because all the receivers arranged along the vertical line of reception analyze similar wave processes. Therefore, we can make the analysis based only on one point corresponding to each vertical line of reception, without a loss of quality.

2. In order to perform a comprehensive analysis, we need to analyze both the horizontal and the vertical component of the velocity, in spite of the fact that the system solved inside the karst cavern describes the propagation of acoustic waves, i.e., only P-waves. The P-waves have a certain direction relative to the line of reception.

3. In order to carry out a comprehensive analysis, we need to describe the seismic wave propagation inside the rock formations by solving the system of equations describing the state of the linear-elastic medium, i.e., consider all types of waves (both longitudinal, transverse, the Rayleigh waves, etc.) in spite of the fact that inside the ravine and karst cavern the system solved describes the acoustic, i.e., only P-waves. This is because under certain conditions, S-waves can produce considerable responses in terms of amplitude in the acoustic medium, and these responses depend on the angle of incidence of the S-waves onto the interface between the linear-elastic and the acoustic media. This means that the majority of effects are lost in the description of rock formations by the system of equations for the propagation of acoustic waves (see points 12, 13).

4. In the case of an explosion in the rock formations, the seismic waves reach the karst cavern sooner than when the explosion occurs at the bottom or in the center of the ravine, in spite of the fact that the distance in meters between the point of he texplosion in the rock formations and the karst cavern is larger than the distances in meters between the karst cavern and the point of the explosion at the bottom of the ravine or at its center. This is due to the fact that the seismic waves generated by an explosion in the rock formation start to propagate in the rock formation immediately, while in the case of an explosion at the bottom of the ravine or a its center, the seismic waves need time to overcome the distance from the point of the explosion to the rock formation.

5. In the case of an explosion at the bottom of the ravine, the seismic waves reach the karst cavern sooner than in the case of an explosion in the center of the ravine, in spite of the fact that the distance in meters between the point of explosion at the bottom of the ravine and the karst cavern is larger than the distances in meters between the karst cavern and the point of the explosion at the center of the ravine. Thist shows that in the case of an explosion at the bottom of the ravine, the seismic waves need less time to reach rock formations than in the case when the explosion occurs in the center of the ravine.

6. For the case when the karst cavern is adjacent to the ravine, the horizontal component of the velocity makes the greatest contribution for all types of explosions, which is stipulated by the direction of the entry of the seismic waves into the karst cavern.

7. If the karst cavern is located at a distance of 200 m from the ravine, the relation between contributions from the velocity's horizontal and vertical components shifts in favor of the velocity's vertical component contribution, which is greater than in the case of the karst cavern adjacent to the ravine. This is stipulated by the direction of entry of the seismic waves into the karst cavern.

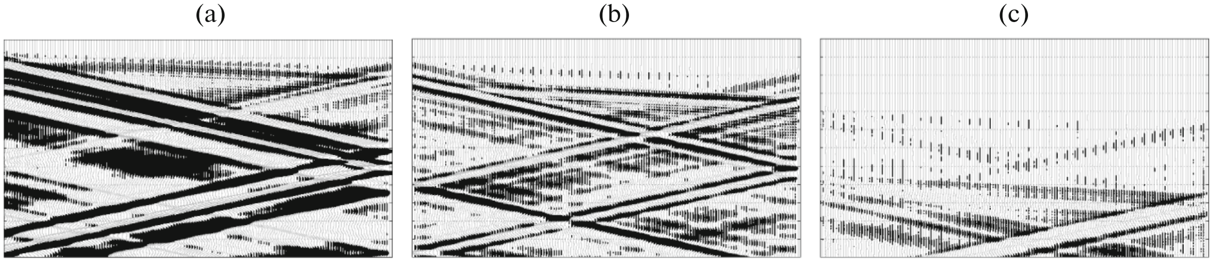

Fig. 6. Seismograms of horizontal component of velocity for case of explosion in rock formations.

8. If the karst cavern is located at a distance of 200 m from the ravine, the relation between the contributions from the velocity's horizontal and vertical components moves even closer to unity. This is stipulated by the direction of entry of the seismic waves into the karst cavern.

9. In the case of explosions at the bottom of the ravine and at its center, the seismograms are qualitatively similar by pattern but are shifted over corresponding instants. The relations between the amplitudes of vhe elocity's horizontal and vertical component vary but maintain the qualitative characteristics of the seismogram's patterns for each component.

10. For the case of an explosion in the rock formations, the seismograms fundamentally differ by the qualitative characteristics of their patterns from the cases of explosions at the bottom and at the center of the ravine due to the qualitative difference between the spatial dynamic wave patterns for these cases. This is stipulated by the fact that the same type of explosion the distance between the karst cavern and the ravine influences the arrival time and amplitude of the transit of different types of waves in different ways.

11. In the case when the karst cavern is adjacent to the ravine, there is the minimum time interval between the arrival of the wavefront at the edge of the karst cavern nearest to the ravine and the time of its arrival at the farthest edge if the explosion occurs in the rock formations.

12. In all cases, the greatest contribution is made by the responses from the S-waves, which originate inside the caverns of rock formations, incident onto the corresponding interface. Then these waves are rereflected inside the karst cavern between the upper and the lower walls, which blurs the contributions from the remaining types of waves.

13. The contribution from the P-waves in the rock formations is minimal due to the spatial shape of the corresponding wavefronts and arrangement of the karst caverns near the daytime surface.

In Fig. 6, seismograms [20] of the horizontal components of the velocity obtained from the horizontal reception lines for the case of an explosion in the rock formations are presented. In Fig. 6a, the case of a karst cavern adjacent to the ravine is presented. In Fig. 6b, the case of a karst cavern located at a distance of 200 m from the ravine is presented, and Fig. 6c shows the case of a karst cavern located at a distance of 1000 m from the ravine.

6. CONCLUSIONS

The spatial dynamic wave processes in rock formations, taking ravines and karst caverns into consideration, have been studied. The results of the study of the dynamics of extending the seismic and acoustic waves generated by different types of explosion sources revealed the typical types of waves depending on the type of explosion, and the laws regulating the waves' spatial dynamic patterns in the rock formations. An examination of seismograms obtained from different reception lines has provided a deeper insight into their behavior depending on the type of explosion and the distance of the karst cavern from the ravine.

The full-wave simulation of the acoustic and seismic wave propagation in the heterogeneous mixed acoustic and the linear-elastic media has been successfully used for the study using the grid-characteristic method that has provided a mathematically and physically correct description of the spatial dynamic wave processes, taking the boundary and contact surfaces, including the boundaries between the linear-elastic and the acoustic media, into consideration.

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REFERENCES

1. A. Bobet, A. Fakhimi, S. Johnson, J. Morris, F. Tonon, and M. R. Yeung, "Numerical models in discontinuous media: review of advances for rock mechanics applications," J. Geotech. Geoenviron. Eng. **135**, 1547–1561 (2009).
2. G. Pande, G. Beer, and J. Williams, *Numerical Methods in Rock Mechanics* (Wiley, New York, 1990).
3. T. Belytschko, M. Plesha, and C. H. Dowding, "A computer method for stability analysis of caverns in jointed rock," Int. J. Numer. Anal. Methods Geomech. **8**, 473–492 (1984).
4. C. H. Dowding, T. B. Belytschko, and H. J. Yen, "Dynamic computational analysis of openings in jointed rock," J. Geotech. Eng. **109**, 1551–1566 (1983).
5. L. E. Schwer and H. E. Lindberg, "A finite element slideline approach for calculating tunnel response in jointed rock," Int. J. Numer. Anal. Methods Geomech. **16**, 529–540 (1992).
6. C. H. Dowding, T. B. Belytschko, and H. J. Yen, "A coupled finite element-rigid block method for transient analysis of rock caverns," Int. J. Numer. Anal. Methods Geomech. **7**, 117–127 (1983).
7. J. Yang, T. Liu, G. Tang, and T. Hu, "Modeling seismic wave propagation within complex structures," Appl. Geophys. **6**, 30–41 (2009).
8. A. V. Favorskaya and I. B. Petrov, "Wave responses from oil reservoirs in the Arctic shelf zone," Dokl. Earth Sci. **466**, 214–217 (2016).
9. A. V. Favorskaya, I. B. Petrov, M. V. Muratov, V. A. Biryukov, and A. V. Sannikov, "Grid-characteristic method on unstructured tetrahedral grids," Dokl. Math. **90**, 781–783 (2014).
10. A. V. Favorskaya, I. B. Petrov, A. V. Sannikov, and I. E. Kvasov, "Grid characteristic method using high order interpolation on tetrahedral hierarchical meshes with a multiple time step," Math. Models Comput. Simul. **5**, 409–415 (2013).
11. V. I. Golubev, I. B. Petrov, and N. I. Khokhlov, "Numerical simulation of seismic activity by the grid-characteristic method," Comput. Math. Math. Phys. **53**, 1523–1533 (2013).
12. K. M. Magomedov and A. S. Kholodov, *Grid-Characteristic Numerical Methods* (Nauka, Moscow, 1988) [in Russian].
13. V. D. Ivanov, V. I. Kondaurov, I. B. Petrov, and A. S. Kholodov, "Calculation of dynamic deformation and distructure of elastic-plastic body by grid-characteristic methods," Mat. Model. **2** (11), 10–29 (1990).
14. A. V. Favorskaya, I. B. Petrov, D. I. Petrov, and N. I. Khokhlov, "Numerical modeling of wave processes in layered media in the Arctic region," Math. Models Comput. Simul. **8**, 348–357 (2016).
15. S. Friend, *Sinkholes* (Pineapple, Sarasota, FL, 2002).
16. K. K. E. Neuendorf, *Glossary of Geology* (Springer, New York, 2005).
17. W. Nowacki, *Teoria Sprezystosci* (PWN, Warsawa, 1970)
18. R. LeVeque, *Finite Volume Methods for Hyperbolic Problems* (Cambridge Univ. Press, Cambridge, 2002).
19. L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics,* Vol. 6: *Fluid Mechanics* (Nauka, Moscow, 1986; Pergamon, New York, 1987).
20. V. I. Golubev, "Technique of visualization and interpretation of full-wave seismic modeling results," Tr. MFTI **6** (1), 154–161 (2014).

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