ISSN 2070-0482, Mathematical Models and Computer Simulations, 2015, Vol. 7, No. 5, pp. 439–445. © Pleiades Publishing, Ltd., 2015.
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Simulation of Seismic Processes Inside the Planet Using the Hybrid Grid-Characteristic Method

V. I. Golubev, I. B. Petrov, and N. I. Khokhlov
Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow oblast, Russia
e-mail: w.golubev@mail.ru; petrov@mipt.ru; k_h@inbox.ru
Received January 15, 2014

Abstract—The problem of the propagation of seismic waves in the Earth is studied. The authors propose a method to simulate numerically dynamic processes based on the solution to determine the system of elastic body equations by a grid-characteristic method on structural curvilinear computational meshes. A set of calculations of the propagation of a perturbation (set as a local extension area) in a layered two-dimensional Earth model are carried out. Wave patterns and characteristics of wave responses are compared to analytical solutions and the published analogous results.

Keywords: global seismic activity, computer simulation, grid-characteristic method, planet model
DOI: 10.1134/S2070048215050051

1. INTRODUCTION

The development of a detailed model of the Earth is among the most important scientific and technical problems, on which scientists have been working for more than 50 years. The first modern model (one-dimensional, spherically symmetric) was constructed in the 1940s by Bullen [1]. With the accumulation of seismic data, this model was further developed to the PREM model [2], IASP91 model [3], SP6 model [4], AK135 model [5], and STW105 model [6]. Heterogeneities with respect to the radially symmetric Earth's model along the radius and in angular directions were revealed in the past 10 years [7–9]. Development of new methods of studying and interpreting the global seismic activity of the planet contributed to a large extent to this fact.

The impossibility of varying the real structure of the geological massif and analyzing differences in the registered signal is a problem of processing seismic data. To avoid this difficulty, numerical calculations of wave fields in geological media with a prescribed internal structure can be performed. Direct simulation of global seismic activity of the entire planet with the construction of synthetic seismograms on a daylight surface can validate the assumptions on the planetary interior or can lead to the development of new technologies of seismic data processing.

Many studies were devoted to the simulation of the propagation of seismic waves in a radially symmetric Earth model [10–13]. The development of high-performance computer systems yielded the computations on models with high-velocity heterogeneities. The numerical simulation of the propagation of seismic waves in the PREM Earth model was performed in [14]. Similar computations in the two-dimensional statement were performed in [15, 16]. The effect of a local mantle upheaval of a fixed length on the registered signal was studied in [17]. However, we note that the wave propagation in only the mantle without taking into account the Earth outer core was considered in [15–17]. It is caused by the application of the polar computational mesh which is clustered near the coordinate origin and contains a singularity in the center. An additional simulated boundary condition was set at the mantle interface with the outer core.

The complication of the models of the planet's structure and the necessity of improving the accuracy of the numerical computations led to the development of new approaches to the computer simulation of dynamic processes. In [18], the Direct Solution Method (DSM) [19] was used to calculate the propagation of seismic waves in the IASP91 model in the presence of upper mantle velocity heterogeneities. The Spectral Element Method [20] was applied to simulate the propagation of seismic waves in a radially symmetric Earth model in [21, 22]. The Chebyshev spectral method was used in [23] to calculate the wave propagation in a radially-symmetric Earth model in spherical coordinates in the presence of upper mantle heterogeneities. However, due to the specifics of the method, the computational domain was limited by 80 degrees in the directions of both angles and by 5000 km in radius. A pseudo-spectral method was used

Characteristics of layers in the two-dimensional Earth model

Number of layer	External radius, km	Internal radius, km	Density, kg/m^3	Velocity of longitudinal waves, km/s	Velocity of transverse waves, km/s
1	6370	5870	4000	5	3
2	5870	5370	4000	10	5.1
3	5370	3000	5000	13	6.5
4	3000	1000	11000	9	0.1
5	1000	0	12000	10.2	3.5

in [24] to simulate the processes in the heterogeneous Earth model in the two-dimensional case. Seismic processes were simulated up to 5315 km, including a part of the outer core. Later, the effect of stochastic velocity heterogeneities on the propagation of seismic waves was investigated in [25]. The application of a polar mesh with a singularity in the center was a significant constraint. Thus, it was impossible to simulate the passage of seismic waves through the inner core. This problem was solved in [26]; although, it was solved for only the acoustic case. The simulation of P-wave propagation in a two-dimensional radially symmetric Earth model was performed. The simulation of the propagation of seismic waves in a two-dimensional Earth model with heterogeneities was performed in [27] using the pseudo-spectral Fourier method [28]. To get over the difficulties associated with the presence of a singular point in the center, an extended scheme was proposed, which modified the procedure of computation of the derivative along the radius. The propagation of seismic waves in a two-dimensional radially symmetric Earth model using the Arbitrary high order DERivatives (ADER) method with application of an unstructured triangular mesh was calculated in [29]. Because the application of this mesh eliminated the problem with a singularity in the center, the complete Earth model, containing both outer and inner core, was used in the calculation. However, a thorough quantitative analysis of the results obtained was not performed in this study, and only qualitative patterns of wave processes were presented.

A method of numerical simulation of the propagation of seismic waves in heterogeneous media is proposed in the present study, which is aimed at calculating global seismic processes in the bowels of an elastic planet. The method is based on the numerical solution to determine the system of elastic body equations by a high order accuracy grid-characteristic method [30–34] on structural curvilinear computational meshes. The use of a set of sewn meshes covering the simulation domain allows overcoming the problem of the calculation of wave propagation through the inner core without introducing simulated boundary conditions. The results of a set of calculations of the propagation of a perturbation (set as a local extension area) in a layered two-dimensional radially symmetric Earth model are presented. The results of computer simulation are compared with the results of the analytical calculations and published analogs.

2. FORMULATION OF THE PROBLEM AND TECHNIQUE OF THE CALCULATION

The authors investigated the propagation of seismic waves in a five-layered two-dimensional radially symmetric Earth model. The characteristics of the layers are given in Table 1. The dependences of medium density and the velocities of propagation of elastic waves on the radius were taken from the PREM model [2].

The following equations of the linear dynamic theory of elasticity are used to describe the state of an infinitely small volume of a linearly elastic medium:

$$\rho \cdot \dot{\upsilon}_i = \nabla_j \cdot \sigma_{ij}, \quad \dot{\sigma}_{ij} = q_{ijkl}\dot{\varepsilon}_{kl} + F_{ij}. \qquad (1)$$

Here ρ is the density of medium, υ_i are the components of the displacement velocity vector, σ_{ij} and ε_{kl} are the components of tensors of Cauchy stresses and deformations, ∇_j is the covariant derivative with respect to the jth coordinate, and F_{ij} is the additional right-hand side. A form of components q_{ijkl} of the fourth order tensor is determined by the rheology of the medium. For the linearly elastic case, we take the following form:

$$q_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}).$$

In this relationship, which generalizes Hooke's law, λ and μ are Lame's parameters, and δ_{ij} is Kronecker's symbol.

The first vector equation in system (1) presented two motion equations, the second equation presented three rheological relationships. The vector of unknown functions, composed of five components, has the following form:

$$\mathbf{u} = \{\upsilon_1, \upsilon_2, \sigma_{11}, \sigma_{12}, \sigma_{22}\}^T.$$

System (1) does not have an analytical solution in the general case. Therefore, it is solved numerically in the present study using the grid-characteristic method. The computational domain, which is a circle of a fixed radius, is covered by the grid nodes, where the values of the vector of unknown functions are calculated. Note that the attempt of covering the computational domain by a single structural curvilinear grid leads to a part of its elements being smaller by factors of hundreds and thousands times than the average size of all the elements. Because this abruptly reduces the maximal allowable time step and considerably increases the computational time, the authors proposed and used the procedure of covering the circle by five bound structural curvilinear grids-sectors. In this case, the values in the corresponding nodes of grids are corrected at each time step to conform to the values in them.

The original system of equations (1) can be written in the follows form [33]:

$$\frac{\partial \mathbf{u}}{\partial t} = \sum_{j=1}^{2} \tilde{\mathbf{A}}_{\mathbf{j}} \frac{\partial \mathbf{u}}{\partial \xi_j},$$

where (ξ_1, ξ_2) are the curvilinear coordinates. After splitting by directions in each time step it is necessary to solve the following system:

$$\frac{\partial \mathbf{u}}{\partial t} = \tilde{\mathbf{A}}_j \frac{\partial \mathbf{u}}{\partial \xi_j} = \mathbf{\Omega}_j^{-1} \mathbf{\Lambda}_j \mathbf{\Omega}_j \frac{\partial \mathbf{u}}{\partial \xi_j}, \tag{2}$$

where $\mathbf{\Lambda}_j$ (diagonal matrix whose elements are eigenvalues) has the following form:

$$\mathbf{\Lambda}_j = \operatorname{diag}\left\{ l_j \sqrt{\frac{\lambda + 2\mu}{\rho}}, -l_j \sqrt{\frac{\lambda + 2\mu}{\rho}}, l_j \sqrt{\frac{\mu}{\rho}}, -l_j \sqrt{\frac{\mu}{\rho}}, 0 \right\}.$$

Here $l_j = \left| w^j \right| = \sqrt{(w_1^j)^2 + (w_2^j)^2}$, and $w^j = \nabla \cdot \xi_j$. After substitution of variables $\mathbf{v} = \mathbf{\Omega u}$ system (2) is split into independent transport equations of the following form:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{\Lambda} \frac{\partial \mathbf{v}}{\partial \xi_j} = 0.$$

To solve this equation, the following fourth order accuracy scheme is used,

$$v_m^{n+1} = v_m^n - \sigma(\Delta_1 - \sigma(\Delta_2 - \sigma(\Delta_3 - \sigma\Delta_4))), \quad \Delta_1 = \frac{1}{24}(-2v_{m+2}^n + 16v_{m+1}^n - 16v_{m-1}^n + 2v_{m-2}^n),$$

$$\Delta_2 = \frac{1}{24}(-v_{m+2}^n + 16v_{m+1}^n - 30v_m^n + 16v_{m-1}^n - v_{m-2}^n), \quad \Delta_3 = \frac{1}{24}(2v_{m+2}^n - 4v_{m+1}^n + 4v_m^n - 2v_{m-2}^n),$$

$$\Delta_4 = \frac{1}{24}(v_{m+2}^n - 4v_{m+1}^n + 6v_m^n - 4v_{m-1}^n + v_{m-2}^n)$$

with the hybridization based on the grid-characteristic monotony criterion

$$\min\{v_m^n, v_{m-1}^n\} \le v_m^{n+1} \le \max\{v_m^n, v_{m-1}^n\}.$$

The following correction is applied in the case of its nonfulfilment:

$$v_m^{n+1} = \begin{cases} \max\{v_m^n, v_{m-1}^n\}, v_m^{n+1} > \max\{v_m^n, v_{m-1}^n\}, \\ \min\{v_m^n, v_{m-1}^n\}, v_m^{n+1} < \min\{v_m^n, v_{m-1}^n\}, \\ v_m^{n+1}, \min\{v_m^n, v_{m-1}^n\} \le v_m^{n+1} \le \max\{v_m^n, v_{m-1}^n\}. \end{cases}$$

Having transported all the components of v, the solution is recovered:

$$\mathbf{u}^{n+1} = \mathbf{\Omega}^{-1} \mathbf{v}^{n+1}.$$

It is necessary to note that the proposed method of calculation is easily generalized to the three-dimensional case. The domain of interest (full sphere) can be covered in this case by seven curvilinear structural grids. However, due to the abrupt increase of the number of nodes, a significant increase in the computational time takes place, which should be compensated by an intensive application of the techniques of parallel computations (MPI and OpenMP).

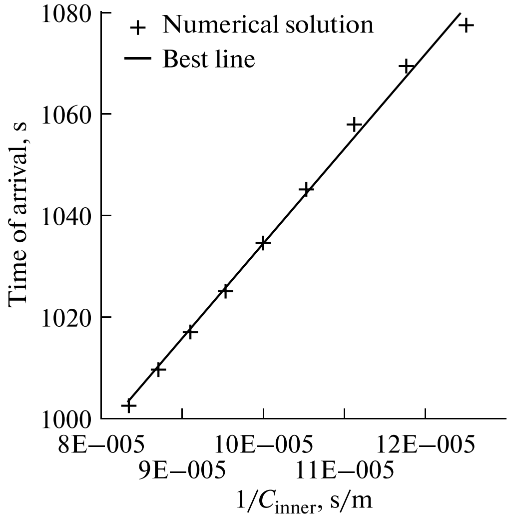

Fig. 1. Time of the first arrival as a function of the velocity of propagation of longitudinal waves in the inner core.

3. EXAMPLES OF COMPUTATIONS

A series of computations with the variation of the parameters of the planet model (radius of the inner core and velocity of propagation of longitudinal waves in it) were carried out at the first stage of the investigation. The model of a radially expanding ring was used as the initial perturbation. The depth of occurrence was 870 km, the external radius was of 500 km, the internal radius was 100 km, and the initial velocity was 1 m/s. The computational grid was composed of ~1 million nodes and the minimal size of a cell was of 15 km. The purpose of this series of numerical experiments was to compare the longitudinal wave traveltime along the planet's principle diameter calculated analytically and that obtained from the analysis of a synthetic seismogram. It is known that in the case of a collection of homogeneous infinite elastic plane layers, the problem of propagation of the longitudinal wave, propagating along the normal to them, can be solved analytically. In the process of direct simulation, it is possible to directly register the first arrival on the opposite (from the source) side of the planet, and it becomes of interest to compare the analytical and numerical estimates. As only the time of the first arrival can be calculated at the fixed characteristics and dimensions of layers, it was decided to perform a series of computations with the variation of the model parameters in a wide range.

First, the case of variation of the velocity of propagation of longitudinal waves in the Earth's inner core was considered (layer no. 5 in our model). The range of the values from 8 to 12 km/s with a step of 500 m/s was studied. A seismometer that registered the vertical component of the medium's shear velocity was located on the opposite (from the explosion) surface of the Earth. Based on its readings, the first arrival of a longitudinal wave was picked and the time of its arrival was measured. In the case of a collection of parallel layers, the time of arrival was determined by the following relationship:

$$T_{\text{arrival}} = \text{const} + \frac{2R_{\text{inner}}}{C_{\text{inner}}},$$

where R_{inner} is the inner core radius and C_{inner} is the velocity of propagation of longitudinal waves in the inner core.

The dependence of the time of arrival of a longitudinal wave on the velocity of propagation of longitudinal waves in the inner core was constructed based on the results of numerical simulation (see Fig. 1). The closest line to the experimental data has a slope coefficient of 1874 km, which coincides with $2R_{\text{inner}}$ = 2000 km with an accuracy of 7%.

After that, the case of variation of the radius of the Earth's inner core was considered. Note that with an increase in the thickness of layer no. 5, the thickness of layer no. 4 decreases by the same magnitude, respectively. The computations were performed for parameter values ranging from 0 to 3000 km with a step of 500 km. In a similar manner to the procedure described above, the time of the first arrival on the opposite side of the planet was registered. In the case of parallel elastic layers, we have the following analytical dependence:

$$T_{\text{arrival}} = \text{const} - 2R_{\text{inner}}\left(\frac{1}{C_{\text{outer}}} - \frac{1}{C_{\text{inner}}}\right),$$

where R_{inner} is the inner core radius, C_{inner} is the velocity of propagation of longitudinal waves in the inner core, and C_{outer} is the velocity of propagation of longitudinal waves in the outer core.

The dependence of the time of first arrival on the radius of the inner core was constructed based on the results of computer simulation (see Fig. 2). The closest line to the experimental data has a slope coefficient of 0.0274 s/km, which coincides with an analytical value of 0.0261 s/km with an accuracy of 5%.

In addition, the results of the numerical computation were compared with the results published in [29]. Unfortunately, the authors had not described in detail the computational experiment, namely, the Earth's velocity model, the parameters of the initial perturbation, and the way of representing wave patterns. The source of perturbation, described previously, was also used in our calculations. Figure 3 shows the wave

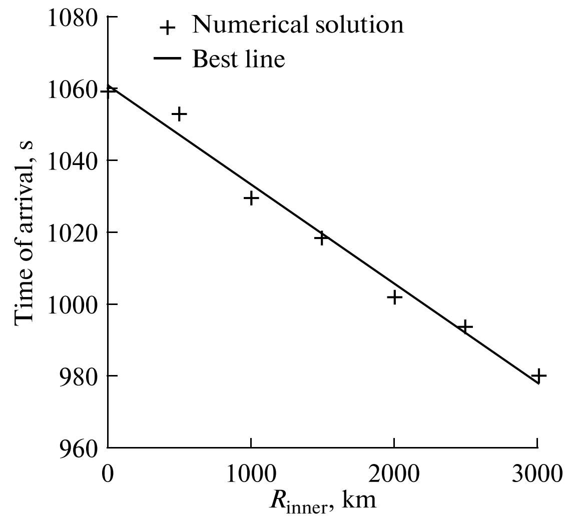

Fig. 2. Time of the first arrival as a function of the radius of the Earth's inner core.

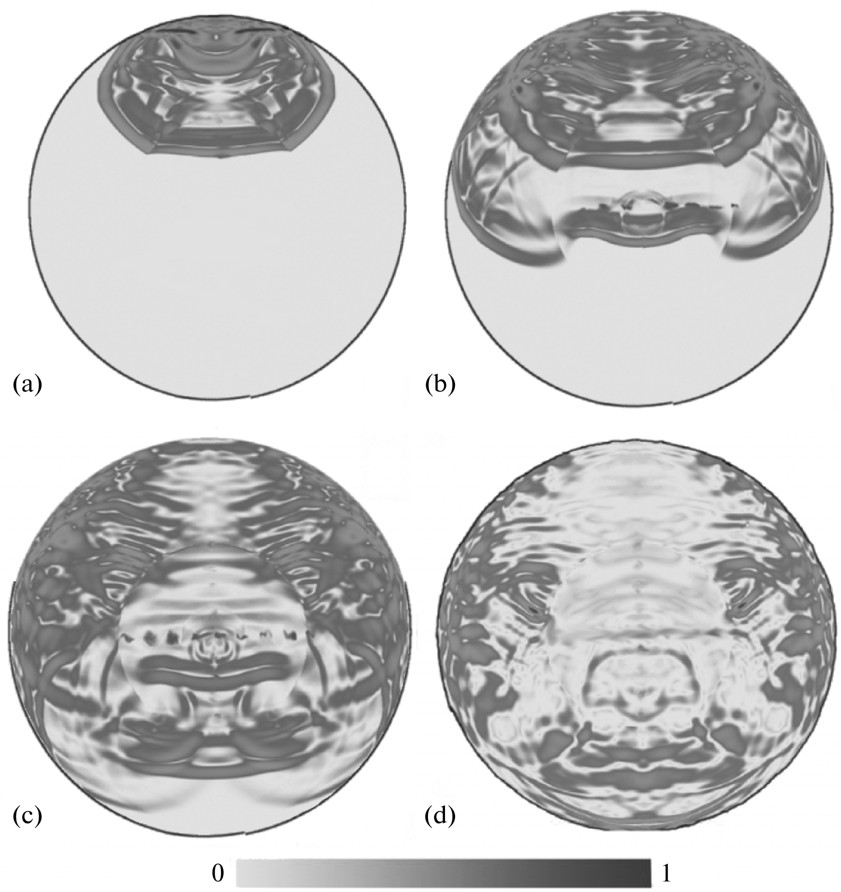

Fig. 3. Distribution of the velocity's absolute value in the earth model at successive time moments: (a) 300 s, (b) 600 s, (c) 900 s, and (d) 1200 s from the start of the computation.

patterns (the absolute value of the velocity in gray), constructed based on the calculated velocity fields, normalized by the signal's maximum, at successive time moments. A qualitative coincidence of the results with the results of [29] is observed: the presence of reflections from the boundaries of the inner and outer cores and wave front bends when passing through a heterogeneous medium. Certain discrepancies can be caused by the difference in the applied models of the initial perturbation and planet structure.

In addition, it is necessary to note that the proposed method of calculation on structural meshes should exceed by velocity the method of calculation on unstructured triangular meshes, due to the absence of the search for adjacent nodes in the computational stencil. As the data on the code's computational time were not published in [29], direct comparison is impossible.

4. CONCLUSIONS

The method of simulation of the propagation of seismic waves in the elastic model of the Earth on structured curvilinear grids was proposed in this study. To overcome the problem with a strong decrease in the sizes of elements when covering the circle by a single grid, it is proposed to use a set of structural grids with common boundaries. This approach makes it possible to calculate the propagation of waves in the interior of the Earth on the model that includes both the outer and the inner cores. The system of hyperbolic-type equations of a linearly elastic body was used to describe the dynamic processes in the medium. The numerical solution is obtained using the grid-characteristic method.

The authors compared the solution with the analytical solution for the case of parallel homogeneous layers. The comparison confirmed the correctness of the calculation of the propagation of a longitudinal wave along the principle diameter of thevplanet. An insignificant quantitative deviation can be associated with the difference in the curvature radius of the interface between neighboring layers from zero in the numerical experiment. In addition, the propagation of elastic waves from the center of expansion in the two-dimensional layered Earth model PREM [2] was simulated. A qualitative coincidence between the results of the numerical computations and the results obtained by other investigators using unstructured triangular meshes [29] was demonstrated. The observed differences can be explained by the discrepancies of the applied models of the planet and initial perturbations, as well as by the peculiarities of visualization of the results.

Because the earthquake's focus is among the natural sources of seismic waves in the Earth's interior, it is promising to solve numerically the problem of global seismic activity with this source. A mechanico-mathematical model of the hypocenter of an earthquake of the "movement from the fault" type was proposed in [33, 34] for both the two-dimensional and the three-dimensional cases. In addition, the seismic processes in bounded geological massifs, including multilayered ones, were simulated in [33, 34]. The extension of the ideas of the mentioned studies to the problem of global seismic activity and the transition to the full three-dimensional formulation of the problem, ensured by the parallelelism of the computational algorithms for the multigrid geometry using the MPI and OpenMP technologies, are seen by the authors as further directions of investigations.

ACKNOWLEDGMENTS

This study was supported by the Russian Foundation for Basic Research, project no. 14-07-31181 mol_a and grant MK-3383.2014.9 of the President of the Russian Federation.

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Translated by A. Ignatyeva